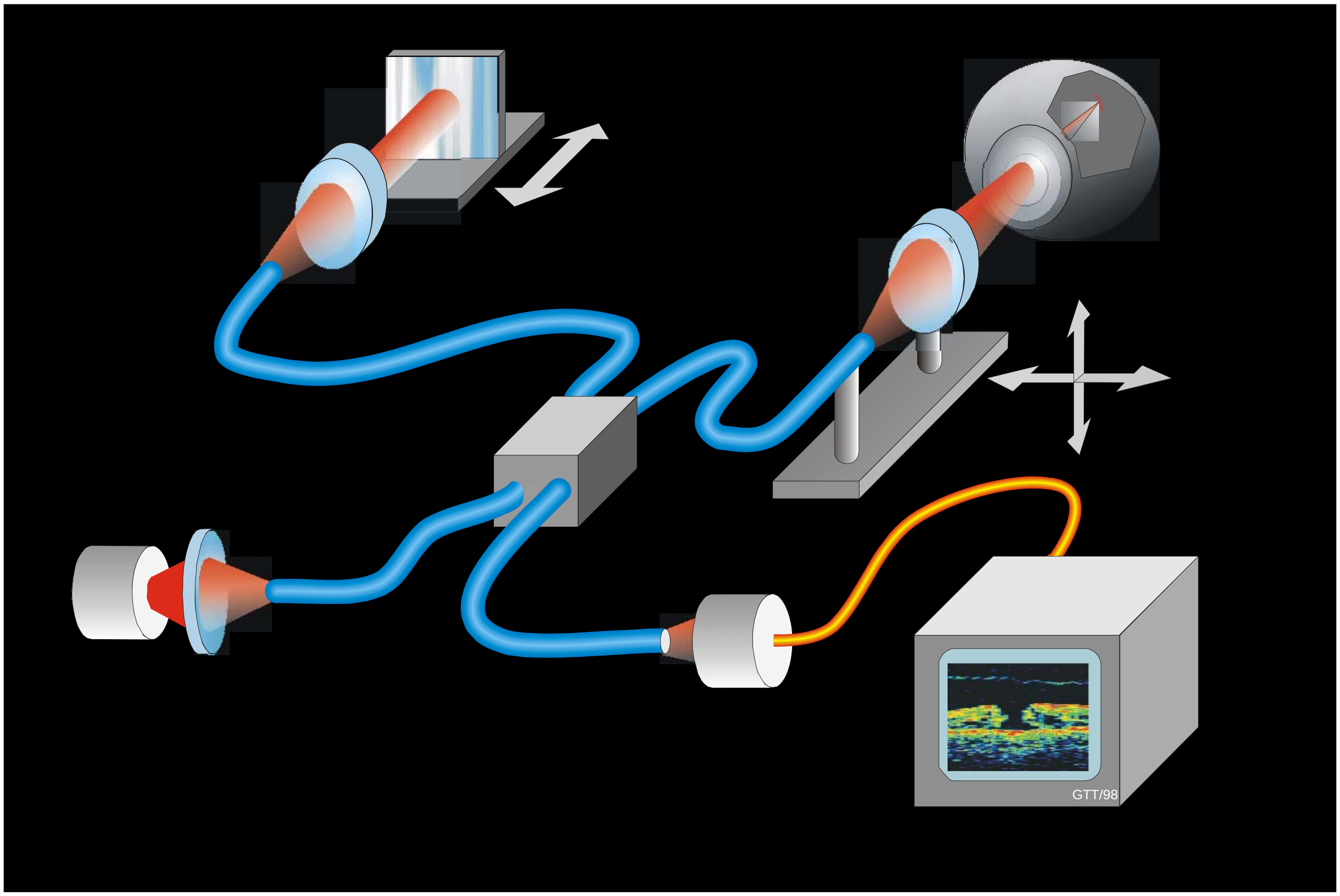
Wave Optics and Lasers



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I-Wave Optics

1. Wave nature of light.

• Water waves & light waves. Light is a form of electromagnetic energy that propagates through space in a wave fashion. It's often helpful to think about the wave properties of light in terms of waves in water. For example, if a pebble is dropped into a still pond, we notice that several evenly spaced wave crests and troughs move outward in a circle from the pebble (Fig. 1). It's important to understand that the water itself is not moving outward, but only up and down. Thus, the water wave is a vertical displacement or "disturbance" of water that travels away from the source.

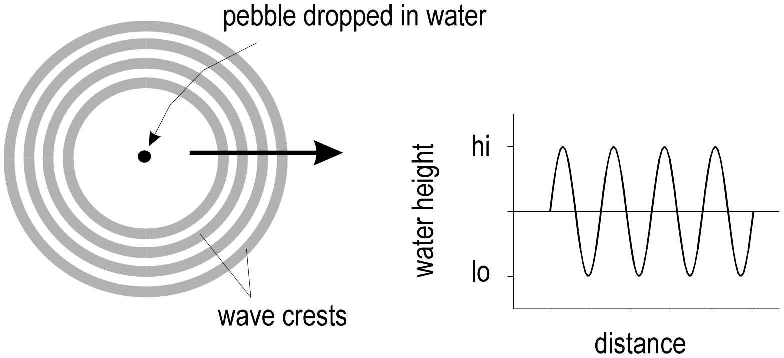

Fig. 1. Left: water waves expanding outward from the impact of a pebble. Right: plot of water height along the line indicated by arrow.

• Why light waves are called *Electro-Magnetic* waves. Light waves are not displacements of matter as are water waves, but rather variations of electric and magnetic fields in space. Electromagnetic waves were first described mathematically by Maxwell[1] who showed that electromagnetic waves have both an electric field (E) and magnetic field (H) that operate in concert. The E-field and H-field are at right angles to each other and are "locked" together so that their amplitudes are both high or low at the same time. The electric and magnetic components of the light wave are complimentary and are not both needed for most description. Consequently, it is customary in wave optics to deal only with the electric field component of the wave and that custom will be observed here.

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[1] James Clerk Maxwell (1831-1879). Scottish physicist, professor of physics at University of Cambridge. In addition to "Maxwell's equations" of electromagnetic radiation, he is also know for many other innovative investigations including the kinetic theory of gases.

- Wavelength and amplitude. Light waves from a point source travel outward with the crests (or troughs) forming concentric spheres around the source, much like the concentric circles in the water wave (Fig. 2). A plot *at a single instant of time* of electric field amplitude as a function of distance along any radial line from the source is shown in Fig. 2. We see that the electric field strength varies sinusoidally with distance. There are two important parameters of this plot. First, the amplitude, *A*, of the wave is the maximum electric field strength. Second, the distance between two peaks (or any two repeating points on the plot) is the wavelength, λ.

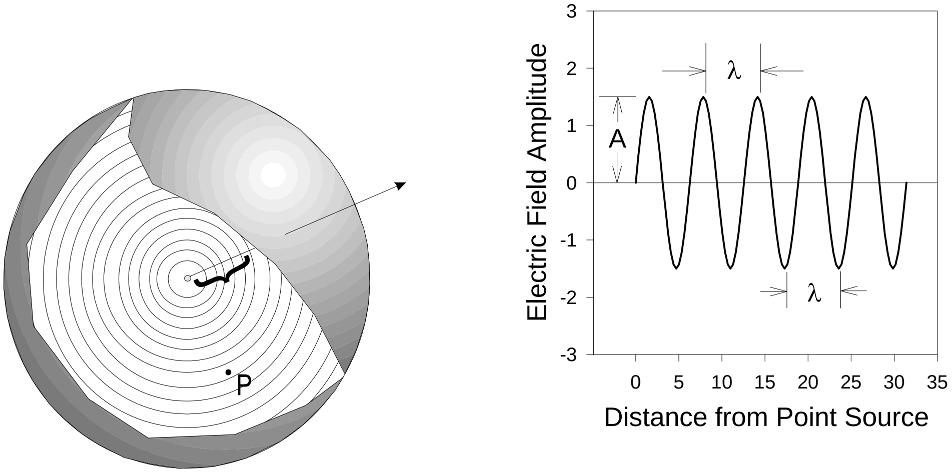

Fig. 2. *Left*: Cut-away portion of outer sphere shows light waves radiating outward from a point source in concentric spheres. *Right*: Plot of E-field amplitude along the radial line in the area indicated by the bracket. Plot is for an instant in time.

The plot in fig. 2 shows the amplitude of the light wave as a function of *distance* from the source at a single instant. We can also look at a light wave as a function of *time*. Suppose, for example, we measure the E-field amplitude at a the fixed point indicated by *P* in fig. 2. The resulting plot of E-field amplitude *vs.* time is shown in Fig. 3. Again, we see that at a single point in space, the E-field amplitude varies sinusoidally in time as the wave passes by.

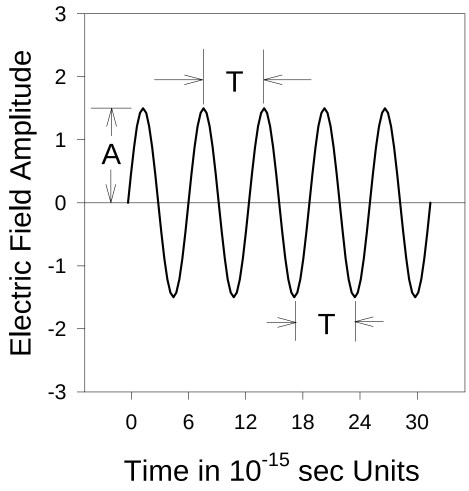

Fig. 3. Plot of E-field amplitude as a function of time at the point "P" shown in fig. 2.

This plot has two important parameters. First is the amplitude of the E field, *A*, as described above. The second parameter is the period, *T*, the time required for one cycle of the wave. The *frequency* of the wave, ν ("nu"), is related to T as

$$T = \frac{1}{\nu}$$

The period, *T*, tells you how long it takes for one cycle of the wave, and the frequency, ν, tells you how many times each second the wave repeats.

Problem 1. What is the period and frequency of a 500 nm light wave in vacuum?

- Speed of light. The speed of light in vacuum is a universal constant, 3×10^8 m/sec. The speed, *c*, is related to the frequency, ν, and wavelength, λ, as

$$c = \nu\lambda$$

It is helpful to think about units to understand this equation. The frequency ν has units of sec^{-1} or equivalently *1/sec* (i.e. "per second"), while the wavelength has units of meters, *m*. Thus,

$$\nu\left(\frac{1}{sec}\right)\times\lambda(m)=c\left(\frac{m}{sec}\right)$$

- Index of refraction. When a light wave enters a medium such as air, glass, or water, the only parameter that doesn't change is the frequency, ν. Both the wavelength and speed change. The ratio of the speed of light in vacuum to its speed in a medium is the *index of refraction, n*.

$$n=\frac{c_{vacuum}}{c_{medium}}$$

You can see that the index of refraction is unitless.

Problem 2. The speed of light in a block of PMMA plastic is measured to be 2.212×10^{8} m/sec. What is the index of refraction of PMMA?

When a light wave enters a medium, not only does its speed change, but also its wavelength, but the frequency, ν, remains constant. How much does the wavelength change? We can calculate this from the equations above as follows

$$\lambda_{vacuum}=\frac{c_{vacuum}}{\nu}$$

$$\lambda_{medium}=\frac{c_{medium}}{\nu}$$

So,

$$\frac{\lambda_{vacuum}}{\lambda_{medium}}=\frac{\left(\frac{c_{vacuum}}{\nu}\right)}{\left(\frac{c_{medium}}{\nu}\right)}=\frac{c_{vacuum}}{c_{medium}}=n$$

$$\boxed{\frac{\lambda_{vacuum}}{\lambda_{medium}}=n\quad Or\quad \lambda_{medium}=\frac{\lambda_{vacuum}}{n}}$$

This equation has the same form as does the relation of *n* to the speeds of light in vacuum and a medium shown above. The equation shows that the wavelength of a light wave in a medium has a shorter wavelength than in a vacuum. The wavelength in the medium is simply the wavelength in vacuum divided by the index of refraction of the medium.

Problem 3. Light from a Helium-Neon laser (λ=633 nm = 633×10^{-9} m) enters the PMMA block in problem 1. What is the wavelength in the PMMA?

- <u>The electromagnetic spectrum</u>. Wavelengths of electromagnetic waves extend from roughly 10^{-16} m to over 10^{6} m, 22 orders of magnitude! It is customary to give names to various bands of wavelengths as shown in the below.

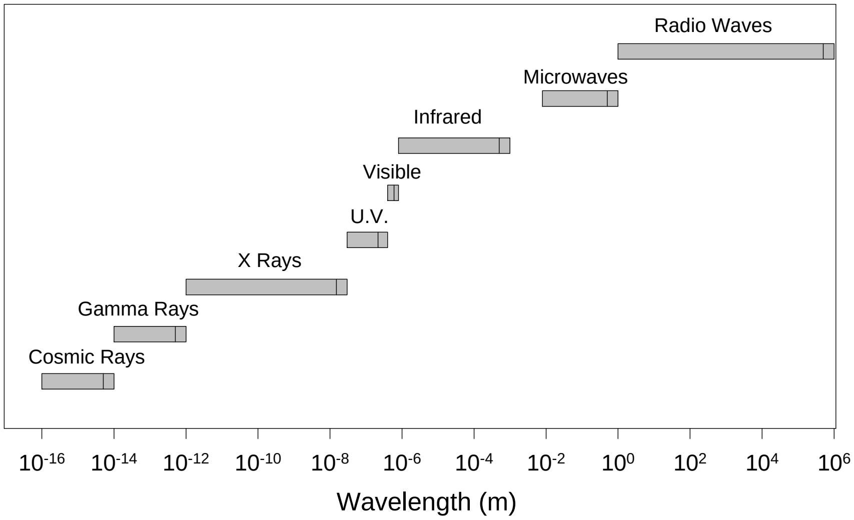

Fig. 4. The electromagnetic wave spectrum.

- <u>Wave-particle duality</u>. The wave description of light cannot account for all observed physical phenomena. For example, in the 19th Century, physicists were unable to account for the observed spectra of glowing heated bodies using wave-optic descriptions. Planck[2] solved this problem by proposing that the energy of electromagnetic waves is directly proportional to their frequency. This is summarized as

$$\boxed{E = \hbar\nu \qquad \hbar = 6.62 \times 10^{-34} \quad joule \cdot sec}$$

where E is the energy (in Joules) of an electromagnetic wave with frequency ν. The quantity $\hbar$ is the proportionality constant that Planck proposed and it is now referred to as “Planck’s constant.” One implication of this equation is that electromagnetic energy is divided into discrete packets or “quanta.” Even Planck didn’t really believe this and it wasn’t until Einstein’s explanation of the photoelectric effect that the notion of quanta began to achieve acceptance.

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[2] Max Planck (1858-1947). Professor of physics at University of Berlin. Awarded Nobel prize in 1918 for his derivation of the law of blackbody irradiation.

The reason this equation is so interesting is that it says if you know the frequency (or wavelength) of a light wave, then you know its energy. For example, what is the energy (*in vacuo*) of a Neodymium:YAG light wave ($\lambda = 1.06\ \mu$m)?

$$\nu = \frac{c}{\lambda}$$

$$\nu = \frac{3 \times 10^{8}}{1.06 \times 10^{-6}} = 2.83 \times 10^{14} \quad sec^{-1}$$

$$E = \hbar\nu$$

$$E = 6.62 \times 10^{-34} \times 2.83 \times 10^{14}$$

$$E = \underline{\underline{1.87 \times 10^{-19} \quad J}}$$

This means that each Nd:YAG "wave" has an energy of 1.87×10^{-19} Joules, or put another way, 1.06 μm light from a Nd:YAG laser comes "packaged" in quanta (photons) each having 1.87×10^{-19} Joules of energy.

Suppose you have developed a surgical technique that requires a burst of Nd:YAG energy of 500×10^{-19} J.

$$\frac{E_{required}}{E_{quatum}} = \frac{500 \times 10^{-19}}{1.87 \times 10^{-19}} = 267.38$$

You cannot make a beam of the required energy because there is no such thing as 0.38ths of the energy of a 1.06 μm Nd:YAG light wave. These photons only come in integral energy multiples of 1.87×10^{-19} J.

2. Diffraction.

- Diffraction and Huygen's principle. "Diffraction" refers to the spreading in space of light wavefronts as they pass through apertures. Diffraction can best be understood using Huygen's principle. Huygens[3] discovered that one could predict the future shape of a wavefront by drawing a series of equally spaced point sources on the wavefront. Each of the point sources emits a spherical wave and by "adding up" the individual spherical waves, you can determine the new wavefront. An example for a portion of a spherical wavefront is shown in Fig. 5.

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[3] Christiaan Huygens (1629-1695). Dutch mathematician, astronomer, and physicist. Founded the wave theory of light, developed new methods of grinding lenses, accurately described rings of Saturn, thought of use of pendulum for controlling clock mechanisms.

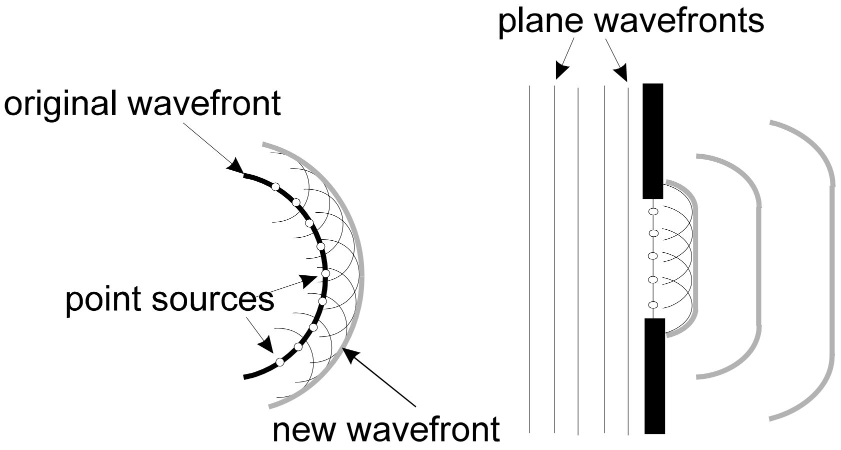

Fig. 5. Left: Illustration of use of Huygen's principle for determining the future position and shape of a spherical wavefront (grey line) from its present position and shape (dark line). Right: Diffraction of plane waves by a small aperture.

Huygen's principle can also be applied to waves passing through an aperture (e.g., a hole in a piece of cardboard) as shown in the right of Fig. 5. In this case the light waves are assumed to be flat (i.e. planar). Using a series of point sources on the plane wave in the aperture, the wavefront exiting the aperture is seen to "bend" backward at the edges. At progressively further distances from the aperture, the wave spreads even further into regions that are usually thought of as "shadow" areas. This spreading of the wavefront *is* diffraction.

- Diffraction by a circular aperture--Airy's disk. A particularly important example of diffraction is illustrated in Fig. 6. There, a plane wave of wavelength λ is shown passing through a pinhole of diameter D. If a screen is placed some distance from the pinhole, we observe a series of concentric bright and dark rings. This diffraction pattern is often referred to as Airy's disk because Airy[4] was the first to describe the pattern mathematically. Airy's derivation is beyond the scope of this work, but in essence it involves mathematically summing the individual wavelets produced by the point sources in the plane of the aperture as shown in Fig. 5.

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[4] Sir George Airy (1801-1892). Astronomer Royal of England from 1835 to 1881.

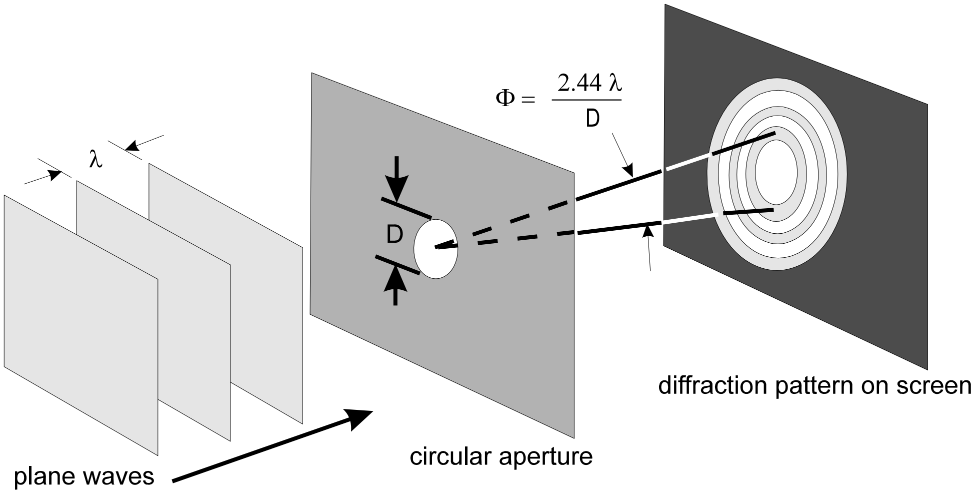

Fig. 6. Schematic of apparatus for producing a circular diffraction pattern (Airy's disk).

If we were to measure the light intensity across the diffraction pattern, we would find the pattern shown in Fig. 7. There is a central bright circle containing about 84% of the light surrounded by a circle of zero intensity. The intensity then increases again, forming a much smaller peak. This pattern repeats as we move radially outward from the center with the peaks (bright circles) decreasing in intensity.

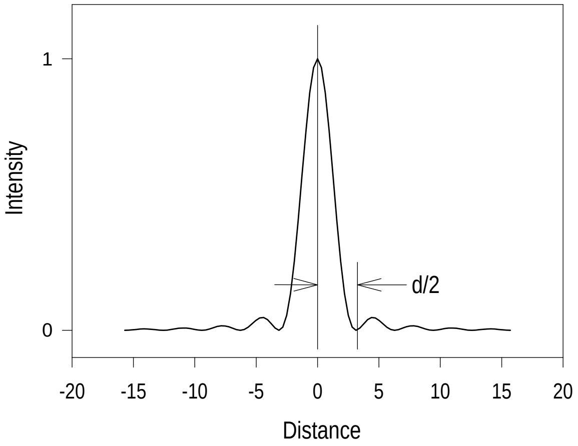

Fig. 7. Light intensity of the Airy's disk diffraction pattern. Distance is in arbitrary units. Zero distance is at the center of the pattern.

A measure of the *amount of diffraction* is the diameter of the first dark ring: the larger the diameter, the more diffraction. As shown in Fig. 6, the angle, *Φ (radians)*, subtended by the first dark ring is

$$\Phi = \frac{2.44\lambda}{D}$$

where D is the diameter of the pinhole. You can see from this equation that as the hole diameter increases, the angle subtended by the first dark ring decreases. In other words, there is less diffraction as the hole becomes larger in diameter. If we want to calculate the diameter in linear units (e.g., mm's) rather than angular units, we need to know the distance from the pinhole to the screen. (For the interested reader, this calculation is shown in Appendix 1.) The answer is

$$d = \frac{2.44l\lambda}{D}$$

where l is the distance to the screen and d is the diameter of the first dark ring. If a lens is placed just in front of the pinhole (or just behind it), the diffraction pattern is produced at the focal length of the lens, f. The length, l, can then be replaced by f, and the resulting formula for the diameter of the first dark ring is

$$\boxed{d = \frac{2.44 f\lambda}{D}}$$

- <u>Diffraction and resolution</u>. Diffracton occurs in all optical systems. The edges of lenses, stops, and other apertures in optical systems are responsible for diffraction. The inherent diffraction of optical systems limits their resolution. The resolution of an optical system can be characterized by the smallest separation between two point sources that would allow them to just be seen as separate. If all other optical aberrations in the system (e.g., coma, astigmatism, spherical aberration) are negligible, then it is the diffraction of the system that limits resolution and the system is said to be "diffraction limited." A common method of characterizing the diffraction limited resolution of an optical system is the Rayleigh criterion. This is the separation two Airy's disks arising from two point sources such that the peak of one Airy's disk falls on the first dark ring of the other (Fig. 8). Since the diameter of the first dark ring in Airy's disk is $2.44\lambda/D$, you can see from Fig. 8 that the two bright peaks of the disks will be separated by 1/2 this amount or $1.22\lambda/D$. So, if we know that an optical system is diffraction limited and we know the wavelength and the diameter, D, of the limiting aperture, then we can calculate the resolution in either angular or linear units.

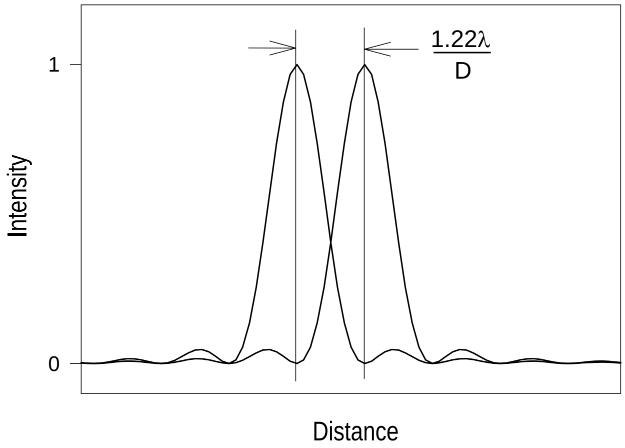

Fig. 8. Airy's disk diffraction pattern from two, just resolvable point sources according to the Rayleigh criterion.

3. Interference.

- Constructive and destructive interference. The phenomenon of interference is the result of the superposition of two or more waves. The principle of superposition states that two (or more) separate waves occupying the same space add up to produce a resultant wave. This is illustrated in Fig. 9, below. In the left panel of Fig. 9, waves A and B are drawn separately for clarity, but imagine they share the same x-axis. The *net* wave resulting from superposition of A and B can be found by adding the Y-axis values of waves A and B. The result is the net wave labeled C. The peaks and troughs of wave C are larger than those of either A or B because the separate peaks (or troughs) add to make a large net peak (or net trough). The two waves *interfere* with each other and produce a larger amplitude net wave, an example of *constructive interference*.

An example of *destructive interference* is shown in the right panel of Fig. 9. Notice that waves D and E are the same wavelength and amplitude, but have been shifted so that the peaks of one correspond to the troughs of the other. Adding the two together produces a net wave of zero as shown in F. The two examples of interference given here illustrate *complete* constructive and destructive interference. When the peaks of one wave do not exactly line up with the peaks or troughs of another wave or if the two waves are of different wavelengths, *partial* constructive and destructive interference occurs.

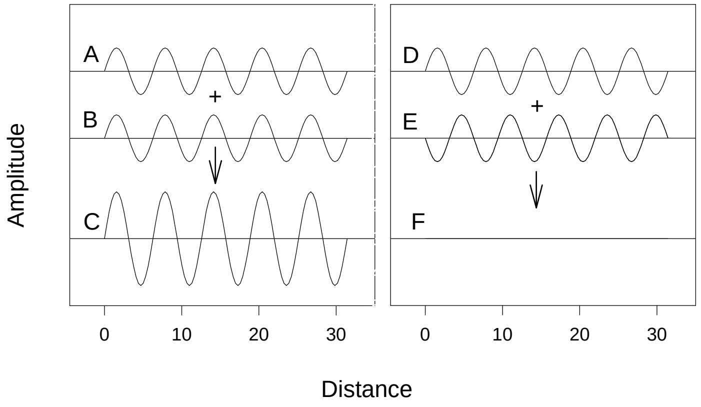

Fig. 9. Illustration of superposition of waves. Left: constructive interference of A and B produces C. Right: destructive interference of D and E produces F, a “flat” wave.

- Light interference: Young’s experiment. There are many examples of interference of light waves, but one of the oldest and most straightforward examples is an experiment performed by Thomas Young.[5] Young’s apparatus is illustrated in Fig. 10. Plane waves from a distant source pass through two very narrow (i.e. somewhat larger than λ) vertical slits in an opaque screen. The two slits are separated by a distance much greater than their widths. Using Huygen’s principle, spherical waves exit from the two slits and radiate outward. The figure illustrates the waves “frozen” in position at a single instant in time. In Fig. 10, the semi-circular lines represent the peaks of the spherical waves. Where the peaks of one wave overlap the other, there will be *total constructive interference* and the light wave amplitude will be higher (i.e. “brighter”). Where the peak of one wave corresponds to the trough of the other wave, there will *be total destructive interference* and the net wave amplitude will be 0 (i.e. “darker”). In between these two extremes, there is *partial interference* (i.e. in between “dark” and “bright”). The rectangular bands drawn over the spherical waves show regions of total constructive and destructive interference. If a screen is placed in the interfering waves as shown in the figure, alternating vertical bands of light and dark stripes will be seen. The bright stripes correspond to the regions of total constructive interference and the dark stripes to regions of total destructive interference.

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[5] Thomas Young (1773-1829). English physician and physicist. Developed the wave theory of light, discovered the phenomenon of light interference, and formed one of the first theories of color vision.

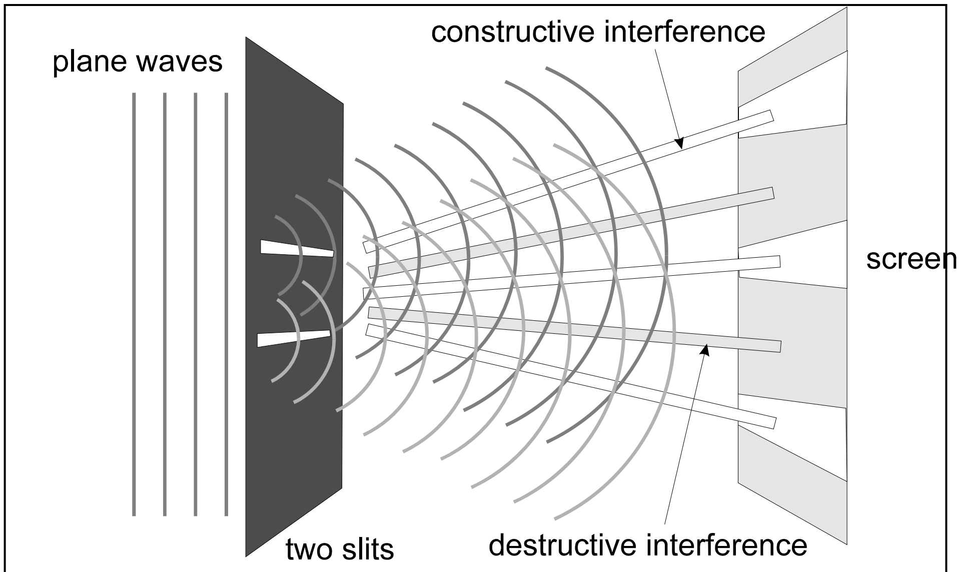

Fig. 10. Illustration of apparatus used by Young to study interference of light waves. Semi-perspective view is looking down on the slits and screen from above.

- Laser speckle and light interference. Another, more commonly observed example of light interference is that of laser speckle. Laser speckle is readily seen, for example, when a laser beam is shone on a wall. Speckle arises because the wall (or most other surfaces) are not perfectly flat, but have microscopic surface roughness. In Fig. 11, a small portion of a hypothetical screen or wall shown at very high magnification reveals the surface roughness. One can think of this rough surface as a series of very small reflectors all at varying angles to one another. A plane wave from a laser falling on the surface is reflected at different angles by each small area, thus each small area acts like a point source producing a spherical wavelet. Where peaks of the wavelets cross, there is constructive interference, producing a bright point or a "speckle." Because the "small reflectors" of the surface can be of varying size and random orientation, the resulting speckle pattern also appears random in the positions and intensities of the speckles.

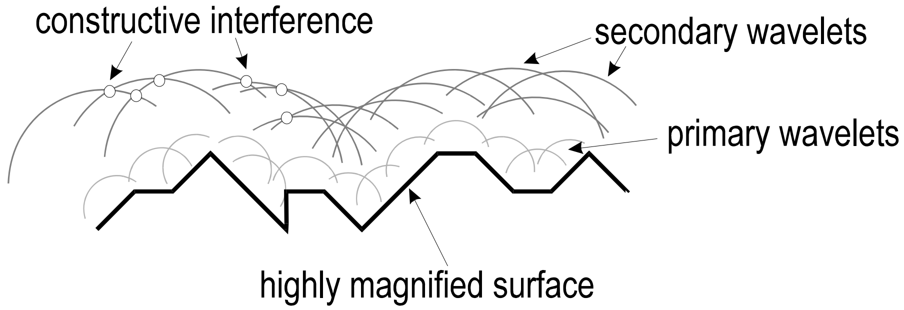

Fig. 11. Formation of laser speckle pattern. The highly magnified surface is illuminated by plane waves from a laser. Reflected wavelets from surface elements interfere in a random pattern.

4. Coherence.

- Coherence and marching. The coherence of a light wave is a measure of how well the wavefronts of the wave stay lined up in space and how repetitive they are in time. A good analogy to a light wave is marchers in a parade as illustrated by the little people below (Fig. 12) in which each line of people represents the crest of a wave. The "people wave" on the left has high spatial coherence: each marcher is in line with others in each crest, and the lines are evenly spaced (i.e., the wavelength is constant). This highly coherent arrangement would allow us to predict the positions of each person and line at any time in the future, that is, the wave is *temporally* coherent as well as *spatially* coherent.

Coherence is a matter of degree, ranging from coherent to partially coherent, to incoherent. The marching group on the right of Fig. 12 illustrates an incoherent wave: the marchers are not lined up and the lines are not separated by equal spaces. It would be impossible to predict future positions. *Partial coherence* is between full coherence and total incoherence as illustrated in Fig. 12. Here the marchers are only approximately lined up in each crest, and the individual lines are only approximately evenly spaced.

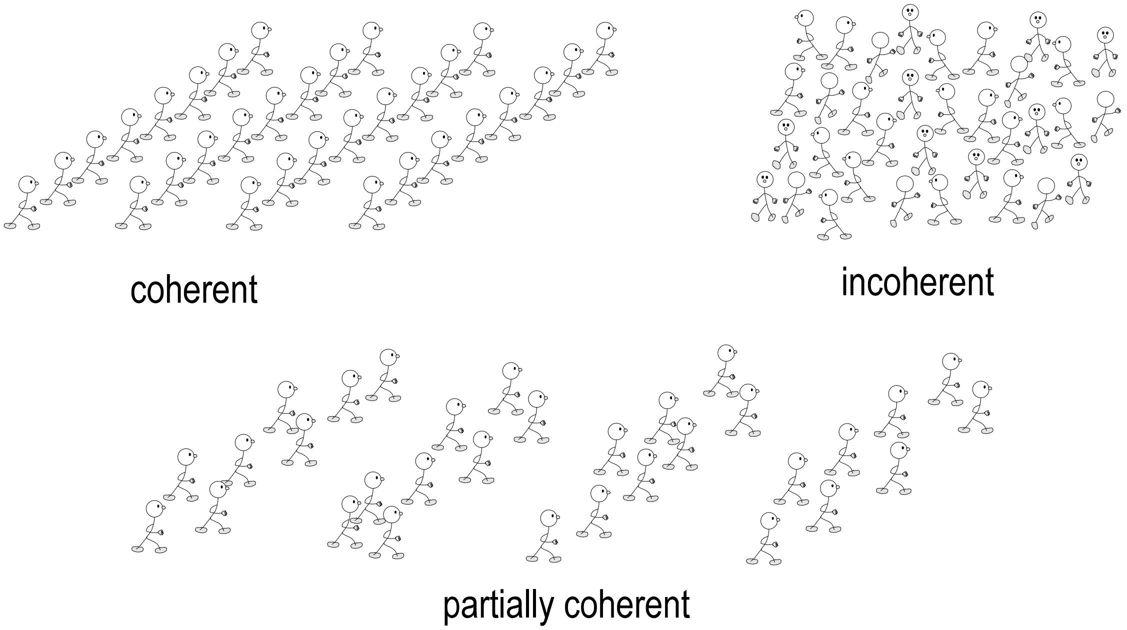

Fig. 12. Illustration of coherence analogous to marchers.

- Coherence of laser light. Light waves from lasers are the most coherent light sources available, but even with lasers, the wave loses its coherence at some point. For example, in the figure below (Fig. 13), the crests of the light waves as they exit the laser are all straight and evenly spaced by the wavelength, λ. Some distance away, however, the waves become "ragged," that is they are no longer in a plane (shown here as a straight line) and their spacing becomes irregular. The distance from the laser over which the waves are coherent is called the *coherence length* of the laser light. A typical mass-produced HeNe laser has a coherence length of about 30 cm. Diode lasers with multiple emitters have extremely short coherence lengths of 1 mm or less.

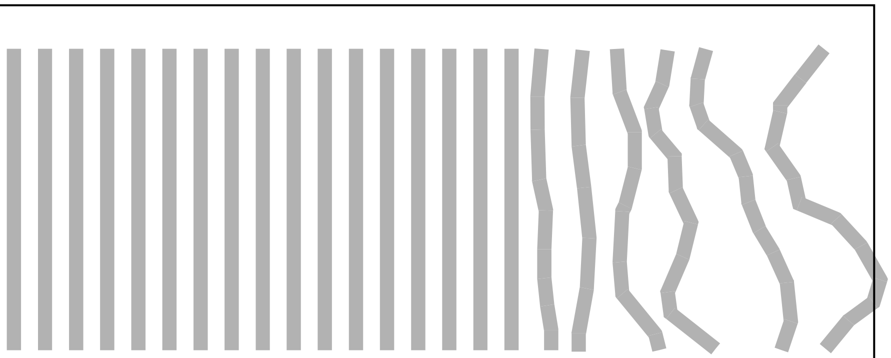

Fig. 13. Coherent plane waves emanating from a laser on the left (not shown). After some distance (and time) waves lose their spatial and temporal coherence.

Although it may seem surprising, *coherence length* is also a measure of the *temporal coherence* of a light source. The two measures of coherence length, l_c, and coherence time, t_c, are related to each other as

$$t_c = \frac{l_c}{c}$$

- Measurement of coherence--interferometers. The coherence lengths of lasers or other light sources are measured using interferometers, for example, the Michelson[6] interferometer. The *spatial* coherence of the light wave could be measured using the Young's two-slit apparatus described above which is another type of interferometer. The central idea of this method is that the wavefront impinging on the slits is not perfectly straight across its entire length. Rather, the wavefront becomes less and less straight (i.e. less and less spatially coherent) toward its edges. Thus, if the slits are quite close as shown below, it is likely that the "straight" (i.e. spatially coherent) portion of the wave will intersect the slits and an interference pattern will be formed on the screen. As the separation between the slits is increased, they intersect the less straight portions of the wavefront and an interference pattern will not be formed. By adjusting the slit separation, we can find the separation at which an interference pattern is just barely formed. This is the spatial coherence length of the wave.

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[6] A. A. Michelson (1852-1931). American physicist. Measured the speed of light and performed many famous experiments on the interference of light. Graduate of U.S. Naval Academy, Professor of physics at University of Chicago. Received Nobel Prize in Physics in 1907.

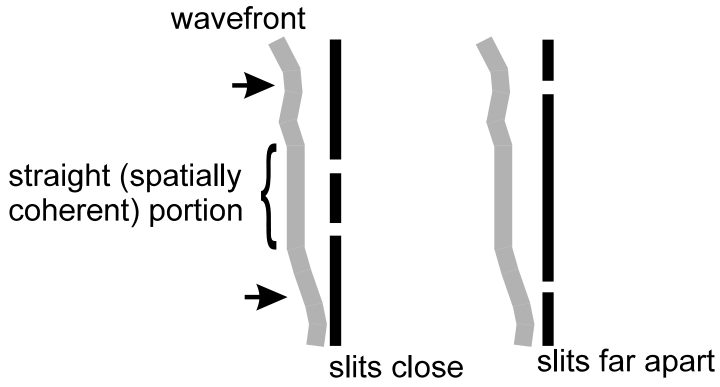

Fig. 14. Finding the spatial coherence length of a plane wave using the Young's 2-slit interference apparatus. When slits are close (left) interference pattern is formed. When slits are far apart, intersecting spatially incoherent part of wave, no pattern is formed.

- Coherence and monochromaticity. One important feature of coherent light is that it is monochromatic, that is, it consists of a single wavelength. Fig. 15 gives a schematic example of the relationship between coherence and monochromaticity. In the figure, a coherent wave is shown originating on the left. The wave gradually becomes temporally incoherent, that is, the wave crests and troughs become unevenly spaced. Notice that the uneven spacing of the wave implies that the wavelength has changed and the wave is now has several different wavelengths, λ, λ_1, λ_2, λ_3. This, in turn, means that the wave is not monochromatic.

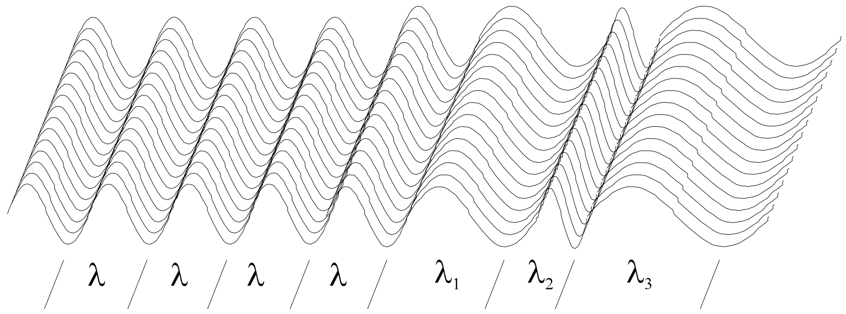

Fig. 15. A light wave originating at left gradually becomes temporally incoherent, thus the wavelength changes, thus it is no longer monochromatic.

5. Polarization.

- Plane polarization. The electric field component of a light wave oscillates in a plane as illustrated in Fig. 16. There, the light wave is shown in an X-Y-Z coordinate system and is propagating in the Z direction. The plane of oscillation makes an angle, θ with the Y axis. The plane of E-field oscillation is the *plane of polarization* of this wave. If we were to look straight down the Z axis, the wave would appear as shown to the right in Fig. 16. The E-field is simply represented by a double arrow that makes an angle θ with the Y axis. The arrow represents a plane polarized light wave.

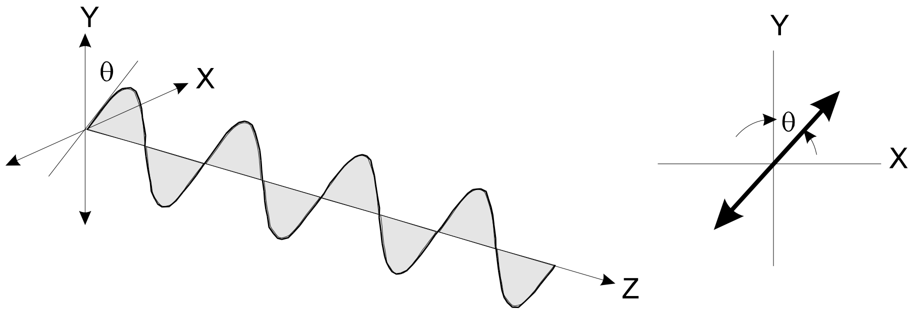

Fig. 16. Left: E-field of a light wave originating at left and traveling in the Z direction. Right: E-field would appear to be a line (double arrows) if viewed by looking back along the Z axis.

- Random polarization. Most light sources we encounter do not produce light waves that all oscillate in a single plane as shown above. Rather, they are made of light waves having many different, random planes of polarization. This state of *random polarization* is illustrated below where many different light waves of different planes of polarization are shown propagating along the Z axis.

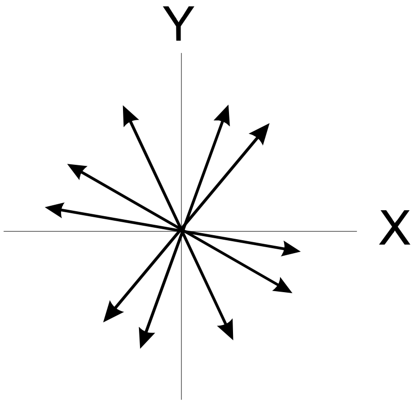

Fig. 17. Randomly polarized light consists of E-field oscillations in many different planes.

- <u>Circular polarization</u>. Some materials cause the X and Y components of a plane polarized light wave to travel at slightly different speeds with respect to each other. The resultant E-field is not in a plane, but rotates in time. The result of this rotation is a circular pattern of the net E-field vectors. This is called *circular polarization*.

- <u>Polarization by selective absorption</u>. Perhaps the most common polarizing material is that found in plastic sheet polarization material such a that used in sunglasses. Such material is made of long molecules oriented in a single direction by heating and stressing the plastic. Conceptually, one can think of the polarizing material as a "picket fence" in which only light waves polarized in the direction of the pickets will pass through. For example, in Fig. 18, the polarizing material is shown as vertically oriented. A light wave that is also polarized vertically will pass through (Fig. 18A), while one polarized horizontally will be absorbed (Fig. 18B). If randomly polarized light impinges on the polarizer, only those waves with more or less vertically polarized E-fields will pass through as shown in Fig. 18C. Roughly 1/3rd of the incident randomly polarized light will pass through the polarizing material making it useful for sunglasses.

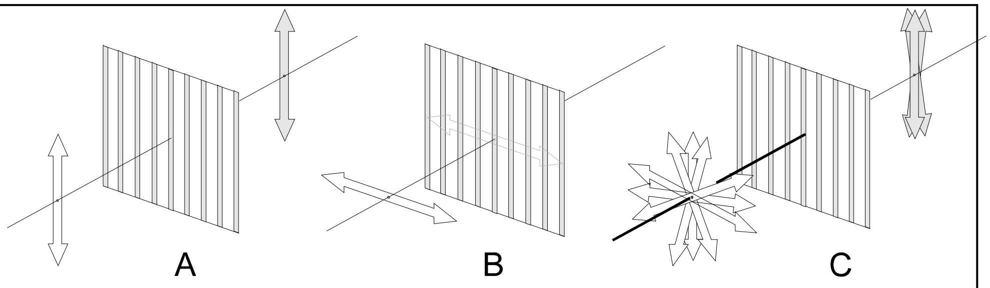

Fig. 18. Plane polarizers. A: Light wave polarized parallel to the polarizer passes through. B. Light wave polarized perpendicular to the polarizer axis is absorbed. C. Components of randomly polarized light parallel to polarizer axis pass through, the rest is absorbed.

- <u>Polarization by reflection</u>. Reflective materials differentially reflect polarized light. The amount of light reflected depends on the orientation of the plane of polarization with the mirror as shown in Fig. 19. This is illustrated below with a light wave that has both horizontal and vertical polarization components. The light wave strikes a tilted piece of glass. The horizontally polarized waves are parallel to the plane of the glass and are strongly reflected. The vertically polarized component, however, is not strongly reflected and is transmitted by the glass plate. This phenomenon depends very strongly on the angle made by the two components with respect to the mirror, indicated by the angle θ in the sketch.

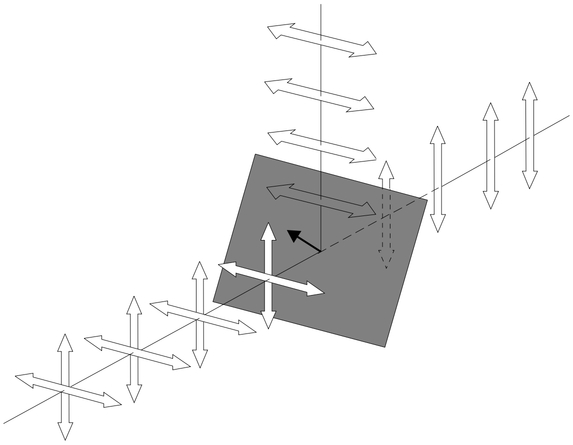

Fig. 19. Polarization by reflection. Unpolarized light originates at left and impinges on flat piece of glass (dark area).

The relationship between the angle of the reflecting glass plate and the intensities of the two reflected polarization components was first described by Fresnel[7] in a set of equations now known as Fresnel's equations:

$$I_{\perp} = \left(\frac{\sin(\varphi - \varphi')}{\sin(\varphi + \varphi')} \right)^{2}$$

$$I_{||} = \left(\frac{\tan(\varphi - \varphi')}{\tan(\varphi + \varphi')} \right)^{2}$$

where $I_{\perp}$ is the reflected intensity of light that is polarized parallel to the surface (i.e. polarized perpendicularly to the plane of incidence), $I_{//}$ the intensity of reflected light polarized perpendicular to the surface, and φ and φ' are the angles of the incident and refracted light. For example, using the relation $n \sin \varphi = n' \sin \varphi'$ and assuming $n=1.0$ and $n'=1.5$, one can calculate φ' for each angle of incidence φ and "plug in" the results to the Fresnel equations. This gives the plot (Fig. 20) below.

Notice in this plot that the intensities of the perpendicular and parallel polarization components are equal for angles of incidence of 0° and 90°. At incidence angle of 0° only about 4% of the light is reflected while for an incidence angle of 90° all of the light is reflected. One important point on this plot is the angle of incidence at which none of the parallel polarized light is reflected. This is known as the Brewster[8] angle and is equal to 56.3°. This is important in the

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[7] Augustin-Jean Fresnel (1788-1827). French physicist and engineer. First to create circularly polarized light, the "Fresnel" lens, and to describe mathematically polarization effects.

[8] Sir David Brewster (1781-1868). Scottish physicist and minister. Invented the kaleidoscope and improved the stereoscope.

construction of lasers which often have two “windows” at either end of the laser cavity oriented at just this angle. These pieces of glass are known as Brewster windows.

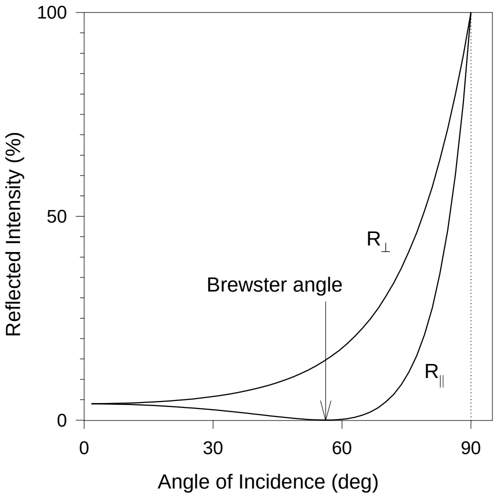

Fig. 20. Plot of intensities of reflected polarized light that is parallel or perpendicular to the plane of incidence as a function of angle of incidence.

- <u>Birefringence</u>. Some materials possess the property of *birefringence*, meaning that they have two different indices of refraction in perpendicular directions. Consequently a light wave oriented parallel to one axis of birefringence will travel at a different speed than a light wave oriented parallel to the other axis. A light wave that forms an angle with both bifrefringence axes is separated into two waves that travel at separate speeds.

6. Scattering and absorption

- <u>Interaction of light and matter</u>. In *absorption*, light energy impinging on a substance is converted to heat, or more specifically to increased rotational and translational movement of atoms and molecules. In essence, one form of energy (electromagnetic) is converted into another (heat). In *scattering*, light energy is absorbed by the substance, but is re-emitted at the same wavelength. This absorption-re-emission comes about because charged molecules of the substance are made to oscillate at the frequency of the light wave. The charged electrons of the oscillating molecules themselves emit light at their frequency of oscillation. The fundamental operations of scattering and absorption are illustrated conceptually in the figure below.

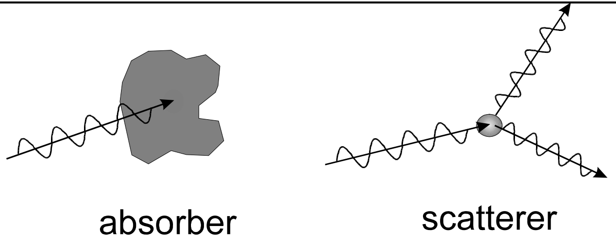

Fig. 21. Left: light impinging on an absorber is converted to heat.
Right: light impinging on scatterer is re-emitted at same frequency.

- <u>Collection of absorbers & scatters</u>. As light passes through materials it is not subject to pure absorption or pure scattering, but a combination of the two. This is illustrated below (Fig. 22) by a box that contains both scattering and absorbing elements. As light passes through the box some is lost due to scattering out of the box and some is lost due to absorption. The intensity of the light leaving the box *I*, is related to the intensity of light entering the box I_0 , by the equation

$$I = I_0 e^{-(\alpha_s + \alpha_a)d}$$

where α_a is the absorption coefficient of the absorbers in units of cm^{-1}, α_s is the scattering coefficient also in units of cm^{-1}, and *d* is the length of the box.

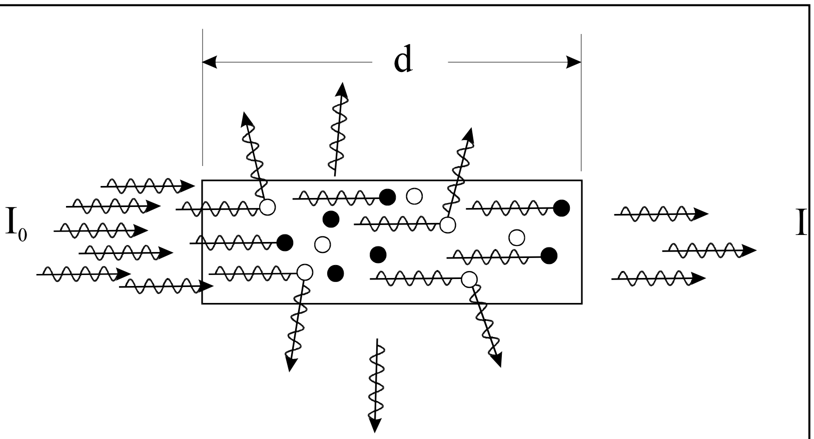

Fig. 22. Light entering a clear box containing scatterers (white blobs) and absorbers (dark blobs). Intensity of entering light is I_0, and intensity of exiting light is *I.*

In any given material α_s may be negligible with respect to α_a or vice versa. For example, a neutral density filter used in optics has a very small scattering coefficient but quite a large absorption coefficient. On the other hand, in biological tissues both α_s and α_a can be significant. For instance, it you point a laser pointer at the skin of your hand you will notice an illuminated area much larger than the laser beam due to tissue scattering.

- <u>Transmittance, absorbance, and optical density</u>. Referring to Fig. 22, above, the intensity of the light entering the medium was I_0 and the light exiting the box had intensity *I*. The *transmittance, T*, is defined as

$$T = \frac{I}{I_0}$$

or, simply put, the fraction of the input light that comes out. The *absorbance* of the medium, *A*, is

$$A = log\left(\frac{1}{T}\right) = log\left(\frac{I_0}{I}\right)$$

The term "absorbance" is used commonly in spectroscopy, but in optics, the same quantity is called the "optical density," *D*.

$$D = log\left(\frac{1}{T}\right)$$

Optical densities are given in "log units." For example, suppose you measure the light intensity entering a piece of dark glass to be 100 mW and the light intensity exiting the glass as 10 mW. The transmittance is

$$T = \frac{10}{100} = 0.1$$

and the optical density is

$$D = log\left(\frac{1}{0.1}\right) = log(10) = 1.0 \quad log\ unit$$

- <u>Scattering and particle size</u>. If white light illuminates very small particles, such as those in wood smoke, the smoke has a bluish appearance. If the particles are larger as in clouds or fog, a whitish appearance is present. Tyndall[9] was the first to investigate experimentally such scattering and it is often referred to as the "Tyndall effect." Later, Rayleigh[10] realized that much scattering was due not to particulates, but to molecules comprising gases themselves. Most importantly, Rayleigh discovered that the intensity of the scattered light in a gas was inversely proportional to the 4th power of the wavelength of the light illuminating the gas. This relationship,

$$I = k\left(\frac{1}{\lambda^4}\right)$$

where *I* is the intensity of the scattered light and *k* is a proportionality constant, is plotted in the figure below. One can see from this plot the dramatic increase in scattering as the wavelength decreases. This is the reason the sky appears blue: short wavelength blue light is more highly scattered toward your eye than are longer wavelengths. It is extremely important to understand that Rayleigh scattering *applies only to particles whose sizes are less than about one tenth the*

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[9] John Tyndall (1820-1893). Irish physicist. Tyndall was a prolific researcher and writer, producing more than 30 books. He succeeded Faraday as superintendent of the Royal Institution. Tyndall is noted for his studies on light scattering, emission and absorption spectrophotometers, the movement of glaciers, and the design of foghorns. He is said to have discovered the effect of penicillin on bacteria.

[10] John William Strutt, third Baron Rayleigh (1842-1919). British physicist. Professor and Chancellor of Cambridge University. Discovered argon for which he received the Nobel prize in 1904.

wavelength of the light. For this reason, Rayleigh scattering is often referred to as “molecular scattering.”

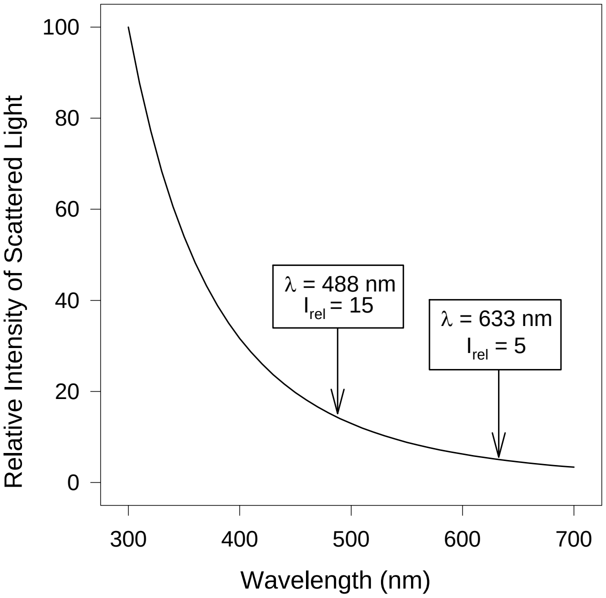

Fig. 23. Intensity of light scattered by particles much smaller than λ as a function of wavelength. Referred to as “Rayleigh” or “molecular” scattering. The relative intensities (I_{rel}) of scattered light from an Argon (488 nm) laser and a HeNe (633 nm) laser are shown.

- <u>Spatial intensity pattern of scattered light</u>. The pattern of scattered light also depends on the size of the scatterers relative to the wavelength of the light. For Rayleigh (molecular) scattering, light is scattered equally in both the forward and backward (toward the light source) directions as shown in Fig. 24. The least amount of scatter is at 90° to the scatterers. In scattering measurements, the intensity at a particular angle (or angles) is usually reported. For example, the (forward) scattered intensity at 45° is shown as the vector in the drawing.

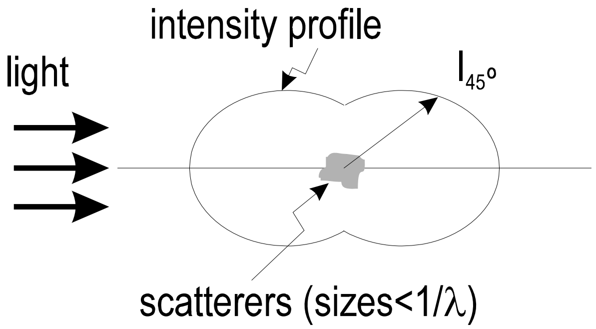

Fig. 24. Intensity profile of Rayleigh (molecular) light scatter.

When the sizes of the scattering particles are not on the order of one-tenth the wavelength of the light, Rayleigh scattering no longer applies and scattering becomes more complicated. There is no sharp size demarcation between Rayleigh scattering and other scattering "regimes," however. When the scattering particles are *spherical and have diameters on the order of* λ, the scattering is sometimes referred to as Mie[11] scattering. Please note, however, that Mie's theory is only one of many that describes scattering by large particles and the particles *must be spheres*. Some authors believe it would be better to refer to Mie scattering as "sphere scattering." The pattern of scattered light is different when the scatterers are large as shown in Fig. 25. In particular, there is much greater scattered intensity in the forward direction than in the backward direction.

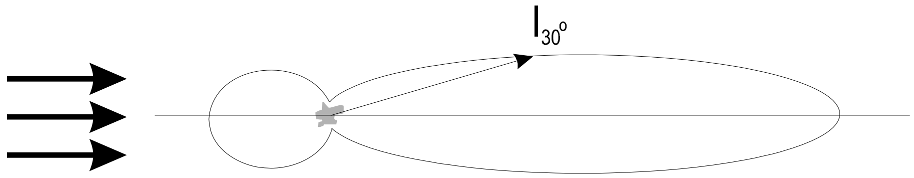

Fig. 25. Intensity distribution of scattered light when the scattering particles are large relative to λ.

• <u>Uses of scattering measurements</u>. Light scattering finds many practical uses in ophthalmology. Qualitative assessment of scattering is valuable in documenting corneal and lenticular clarity. Retinal edema can sometimes be appreciated by viewing the retina with slit illumination and attending to the light that spreads laterally from the slit due to scatter. Other, more experimental, techniques that measure ocular scattering include (a) laser Doppler velocimetry of retinal blood flow, (b) measurement of "haze" following PRK, and (c) "flare" measurements in the aqueous. Scattering also has an important effect on vision. Any scatterers in the optical path of the eye produce some forward scattering (more if the particles are large). The scattered light does not travel to the part of the retinal image it was supposed to and consequently acts like a "veiling" source added onto the retinal image. This reduces image contrast and degrades visibility.

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[11] Gustav Mie (1868-1957). German physicist, professor of physics at University of Freiburg.

II Measurement of Light: Radiometry and Photometry

1. Radiometry and photometry.

Radiometry is the measurement of electromagnetic radiation. As in many other physical measurements, the fundamental units of radiometry are energy (Joules) an power (Watts). Radiometric measurements can be performed on any part of the electromagnetic spectrum. *Photometry* is a special subdivision of radiometry in which measurements are made only on *light visible to humans*.

2. Radiometric units.

• Power. In most cases, the fundamental measurement is the power, P, of a light source. This type of measurement is illustrated below in Fig. 26. Here, a light source emits a cone-shaped light beam. We place a calibrated photodetector in the beam so that all of the light falls on the detector surface. The meter attached to the detector is calibrated in Watts.

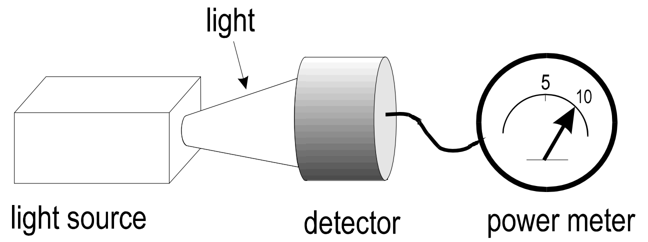

Fig. 26. Apparatus for measuring the power of a light beam.

• Irradiance. Irradiance tells us about the *power density* of our light source if we allow the light to fall onto a surface. In Fig. 27, the light source from Fig. 26, above, is shone on a screen. It illuminates a circular area of radius, r. The irradiance, E, is simply the power of the beam divided by the illuminated area:

$$E = \frac{P}{A} \quad \left(\frac{Watts}{m^2}\right)$$

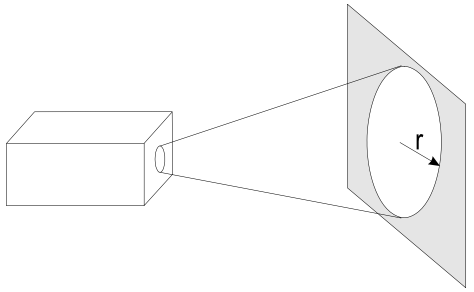

Fig. 27. Irradiance refers to the power per unit area of light falling on a surface. Units are Watts/m^2.

•Radiant intensity. Many light sources radiate in all directions, so we can think of the light passing through a sphere enclosing the source. Radiant intensity tells us how much power passes through a unit portion of the sphere. To illustrate, the light source shown in the figures above is now at the center of a sphere in Fig. 28. The beam from the source intersects the sphere, forming a circular section with area *A*. *The solid angle* corresponding to the area is

$$\Omega = \frac{A}{r^2}$$

The units of solid angle are *steradians*, which comes from the words *stereo*, meaning "solid" and *radian*, a unit of angular measure. The entire sphere subtends a solid angle of 4π steradians. (You can see this if you substitute the expression for the area of the surface of a sphere $(4\pi r^2)$ into the equation above.) Since we previously measured the power of the beam coming from the light source, we can now calculate the radiant intensity as

$$I = \frac{P}{\Omega} \quad \left(\frac{Watts}{steradian}\right)$$

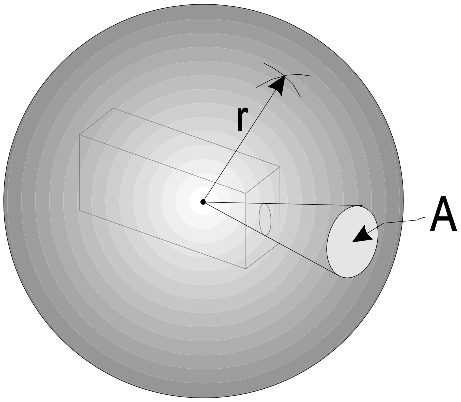

Fig. 28. Radiant intensity is the power of a light source per unit solid angle (steradian).

- Energy. With short pulses of light such as those from some lasers, power (energy per unit time) is not the most appropriate measure. Instead, the energy per pulse is usually given. For example, in Nd:YAG posterior capsulotomy, one keeps track of the energy per pulse which is typically ~2 mJ. The pulse duration is about 10 nS, so the power would be

$$P = \frac{2 \times 10^{-3}\, J}{10 \times 10^{-9}\, sec} = 0.2 \times 10^{6}\, W = 0.2\ Megawatts\ \ !$$

3. Photometric units.

Photometric measurements are exactly analogous to radiometric measurements, but are weighted according to the average spectral sensitivity of the human eye. This weighting function is known as the spectral (or "standard") luminosity curve and expresses the relative sensitivity of the human visual system to different wavelengths of light. The photopic spectral luminosity function is shown in Fig. 29. You can see from this plot that the human visual system is most sensitive to light of about 555 nm (bright yellow). The eye is insensitive to light whose wavelengths are greater than about 700 nm (deep red) or shorter than about 400 nm (deep blue). Thus, it doesn't make any sense to specify a photometric measure of 800 nm light from a diode laser. Furthermore, it doesn't make any sense to give a photometric measure of light for a lizard, for example, unless you know the average spectral sensitivity function of lizards! Since photometric units tell us about how light relates to human perception, they are useful in specifying stimuli in visual psychophysics such as acuity measurement, perimetry, and color vision testing.

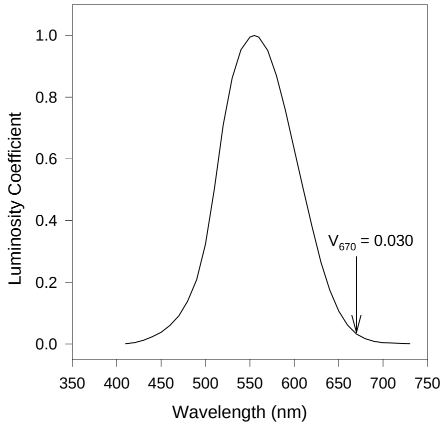

Fig. 29. The human photopic spectral luminosity function.

- <u>Relationship between radiometric and photometric quantities</u>. The fundamental relationship between photometric and radiometric units relates *radiant power* (in Watts) to *luminous power* (in Lumens).

$$F(lumens) = 685 \sum_{400}^{725} P_{\lambda} V_{\lambda} \Delta\lambda$$

where P_{λ} is the power (Watts) at the wavelength λ, and V_{λ} is the luminosity coefficient at that wavelength. This expression is for the case in which there are many different wavelengths making up the light source. It is simpler when there is only one wavelength as with a laser or other monochromatic source. For example, suppose we want to know the luminous power of a "red" laser pointer. We measure the power of the beam using a radiometer and find it to be 2 mW. The wavelength of the beam is $\lambda = 670$ nm. Referring to the luminosity function in Fig. 29, we find that $V_{670} = 0.03$. So, the luminous power, F is

$$F = 685 \sum 2 \times 10^{-3} \times 0.03$$

$$F = 685 \sum 6 \times 10^{-5}$$

$$F = 685 \times 6 \times 10^{-5} = 0.0411 \quad lumens$$

Once we know the luminous power of the source in lumens, we can determine quantities directly analogous to irradiance and radiant intensity, namely *illuminance* and *luminous intensity.*

- Summary of radiometric and photometric units.

Radiometry			**Photometry**		
Term		Units	Term		Units
Radiant power	P	Watt (W)	Luminous power	F	Lumen (Lum)
Irradiance	E	W/m^2	Illuminance	E	Lum/m^2
Radiant intensity	I	W/sterad	Luminous intensity	I	Lum/ster.
Radiance	L	W/m^2/ster	Luminance	L	Lum./m^2/ster.